On the universal tachyon and geometrical tachyon

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ABSTRACT: We study properties of non-BPS D$(p+1)$-brane in the background of k NS5-branes, with one transverse direction compactified on a circle, from the point of view of Dirac-Born-Infeld action. We present the analysis of two different embedding of non-BPS D$(p+1)$-brane in given background and study the classical solutions of given world-volume theory. We argue for the configuration of a non-BPS D$(p+1)$-brane which allows us to find solutions of the equations of motion that give unified descriptions of G and U-type branes.

KEYWORDS: D-branes, Tachyon Condensation.

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Contents

1. Introduction

Non-BPS branes are important and useful objects in string theory [1]. The BPS D-branes can be seen as some kind of soliton solutions on the worldvolume theory of non-BPS branes. Type IIA/IIB string theories contain odd/even unstable non-BPS branes in its spectrum. The most important feature of the non-BPS D-branes is that it contains in its open string spectrum a single mode of negative mass^2 particle in addition to an infinite number of mass$^2 \geq 0$ modes. Unlike the BPS Dp-brane the non-BPS brane is neutral under the $(p+1)$ form gauge field. The study of the physics of tachyonic mode has been one of the most interesting and wide spread subject in the last few years (for a detailed review and for a comprehensive list of references see [2]). In the study of tachyon dynamics on the D-branes the Dirac-Born-Infeld (DBI) analysis surprisingly captures well many aspects of the full theory itself [5 − 10].

However there has not been any clear cut understanding of the geometric picture of these open string tachyonic modes. Not too long back Kutasov [3, 4] gave a geometric interpretation of perturbative open string tachyon in a D-brane system with a different kind of instability. This involves a system of coincident k number of NS5-branes and a BPS Dp-brane placed at a distance opposite to the NS5-branes. This configuration is non-supersymmetric as the NS5-brane and D-brane break different halves of the type II supersymmetry. Many aspects of the dynamics of this system have been investigated in (see for example [11 − 20] and for recent review [21].). Particularly in [4] it was observed that the dynamics of Dp-brane in the background of k NS5-branes on a transverse $R^3 \times S^1$ is remarkably similar to that of the BPS and non-BPS branes in ten dimensional spacetime. It was found out that the object which looks like BPS and non-BPS branes to a six dimensional observer is the same BPS Dp-brane wrapped or unwrapped around the extra S^1. Various interesting facts about the non-BPS D-brane in the background of NS 5-brane wrapped along the transverse $R^3 \times S^1$ has been discussed in detail in [16].

Recently Sen [22][1] argued that under certain conditions these BPS Dp-branes with geometric instability due to its placing at a saddle point of the potential are identified with a non-BPS D-brane with the usual open string tachyon instability thereby giving an interesting geometric meaning. The non-BPS D$(p + 1)$-branes considered in [22] are the ones where the worldvolume directions are along the transverse circle (either wrapping it or ending on NS 5-brane) and along the directions parallel to the Dp-branes in the NS5-brane background. Clearly these D-branes are unstable because of the usual tachyon which lives in their worldvolume. For a large radius of the transverse circle S^1 the descent relation among various unstable Dp-brane system has been discussed in detail. In particular two types of D-branes, namely the G-type D-brane and U-type D-brane seems to be interesting indeed. The G-type of branes have the 'geometrical' instability [4] are obtained by placing the usual BPS Dp-brane ($p \leq 5$) along $x^0, \cdots, x^p$ at $\vec{r} = 0, y = \pi R$ and arbitrary values of $x^{p+1}, \cdots x^5$.[2] The non-supersymmetric U-type D-branes are obtained by placing the non-BPS D-brane along $x^0, \ldots, x^p, y$ directions (where y is the compactified transverse directions of the NS 5-brane). They carry the 'usual' open string tachyon. However it has been found out in [22] that for the large value of the radius of the transverse circle, both G-type and U-type D-branes are actually two phases of the same underlying theory-the worldvolume theory of U-type D-brane.

In the present paper, we would like to investigate the recent conjecture of Sen from the view point of Dirac-Born-Infeld (DBI) analysis. We present two complementary descriptions of a non-BPS Dp-brane. The first one is a direct DBI analysis of the configuration studied recently in [22]. Explicitly, we present DBI analysis of the non-BPS D$(p + 1)$-brane that wraps the transverse circle. We will argue for the existence of classical tachyon solutions that reproduce the D-brane configurations studied recently in [22].

The second approach is based on the analysis presented some time back in [16]. We show that non-BPS D$(p + 1)$-brane that is stretched along the world-volume directions of NS5-brane contains in its solution the non-BPS D$(p + 1)$-brane that wraps the transverse circle and also contains a solution which corresponds to G-type D-brane which sits at the point $Y = \pi R$. We can find these solutions very easily using the mapping of the mode that parameterises the position of a non-BPS D$(p + 1)$-brane along y-direction to the new "geometric" tachyon field $\mathcal{T}$ [3, 4] and then using the profound analysis pioneered in [24]. We believe that the result presented in this paper is the first indication of the unified description of the G and U-branes using the world-volume theory on non-BPS D$(p + 1)$-brane that is localised on y-circle.

This correspondence can be seen more clearly when we perform zero radius limit analysis following [4, 22]. Explicitly, we show that for $k = 2$ (k is the number of NS 5-branes) the geometric tachyon $\mathcal{T}$ and the open string tachyon T have completely the same dynamics. Since these two type of D-branes arise as solutions of the same theory they, from the point of view of this theory, are indistinguishable, which is identical to Sen's observation. In other words, for $k = 2$ the unstable D$(p + 1)$-brane wrapping the transverse circle and

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[1]See also [23].

[2]In the next section we will present the notational details.

the G-type Dp-brane sitting at the point $\tilde{Y} = \pi$ can be interpreted as the same object in the zero radius limit. This can be thought of as a support of Sen's recent conjecture [22].

Rest of the paper is organised as follows. In section 2, we present various unstable D-brane configuration in NS 5-brane background from the view point of Dirac-Born-Infeld analysis. We show the existence of classical tachyon solutions which correspond to various unstable D-brane configurations. In section-3, we study the non-BPS D$(p + 1)$-brane extended along the worldvolume directions of NS 5-branes. We find out a unified description of the G-type and U-type D-brane configurations by using the worldvolume theory on the non-BPS D$(p + 1)$-brane in NS5-brane background that as opposite to Sen's approach is localised at the point $Y = \pi R$. Section 4 is devoted to the study of the correspondence in the zero radius limit. We again find that the non-BPS D$(p+1)$-brane localised at the point $\tilde{Y} = \pi$ gives unified and natural description of U and G-unstable D-branes. We finish the paper in section 5 with conclusions and discussions.

2. Unstable D-brane configurations from DBI point of view in NS5-brane background

Let us consider a system of k NS5-branes in type IIA/IIB theory stretched along $(x^0, \ldots, x^5)$ plane and placed at $(x^6, \ldots, x^9) = (0, \ldots, 0)$. Let x^6 be a compact coordinate with the periodicity $2\pi R$. The string frame metric, the dilaton Φ and the NS sector 3 form field strength H produced by this background are given by[3]

$$
\begin{aligned}
ds^2 &= \eta_{\mu\nu} dx^\mu dx^\nu + H(\vec{r}, y)(dy^2 + d\vec{r}^2) \,, \\
e^{2\Phi} &= g^2 H(\vec{r}, y) \,, \\
H_{mnp} &= -\epsilon_{mnpq} \partial^q \Phi \,,
\end{aligned}
\tag{2.1}
$$

where μ, ν run from 0 to 5; m, n, p, q run from 6 to 9

$$
\vec{r} = (x^7, x^8, x^9) \,, \quad y \equiv x^6
\tag{2.2}
$$

and

$$
H(\vec{r}, y) = 1 + \frac{k}{2Rr} \frac{\sinh(r/R)}{\cosh(r/R) - \cos(y/R)}, \quad r \equiv |\vec{r}| \,.
\tag{2.3}
$$

It is easy to see that the background above is invariant under the following transformations

$$
\sigma : y \to -y, \quad \vec{r} \to -\vec{r} \,.
\tag{2.4}
$$

If the k 5-branes are not coincident but are placed at different points $(\vec{r}_i, y_i)$ $(1 \le i \le k)$ then the solution is still described by (2.1) but with the H given by

$$
H(\vec{r}, y) = 1 + \sum_{i=1}^{k} \frac{1}{2R|\vec{r} - \vec{r}_i|} \frac{\sinh(|\vec{r} - \vec{r}_i|/R)}{\cosh(|\vec{r} - \vec{r}_i|/R) - \cos((y - y_i)/R)} \,.
\tag{2.5}
$$

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[3]We use $\alpha' = 1$ in this paper.

Let us now consider a non-BPS D($p + 1$)-brane embedded in this background. Recall that the action for this system is [5 – 10]

$$S = -\sqrt{2}\mathcal{T}_{p+1} \int d^{p+2}\xi e^{-\Phi} V(T) \sqrt{-\det \mathcal{G}} \, , \qquad (2.6)$$

where

$$\mathcal{G}_{\alpha\beta} = g_{MN} \partial_\alpha X^M \partial_\beta X^N + b_{MN} \partial_\alpha X^M \partial_\beta X^N + \partial_\alpha T \partial_\beta T \, , \qquad (2.7)$$

and where now $\{\xi^\alpha\}$ $(0 \le \alpha \le p + 1)$ are the D($p + 1$)-brane world-volume coordinates and X^M parameterise embedding of D($p + 1$)-brane in target space where $M, N = 0, \ldots, 9$. Finally $\mathcal{T}_{p+1}$ is BPS D($p + 1$)-brane tension.

Note that the equations of motion for X^M that follow from (2.6) take the form

$$\partial_M e^{-\Phi} V(T) \sqrt{-\det \mathcal{G}} + \qquad (2.8)$$
$$+ \frac{1}{2} e^{-\Phi} V(T)(\partial_M g_{NK} + \partial_M b_{NK}) \partial_\alpha X^N \partial_\beta X^K \left(\mathcal{G}^{-1}\right)^{\beta\alpha} \sqrt{-\det \mathcal{G}} - $$
$$- \partial_\alpha [e^{-\Phi} V(T) g_{MN} \partial_\beta X^N \left(\mathcal{G}^{-1}\right)^{\beta\alpha}_S \sqrt{-\det \mathcal{G}}] - $$
$$- \partial_\alpha [e^{-\Phi} V b_{MN} \partial_\beta X^N \left(\mathcal{G}^{-1}\right)^{\beta\alpha}_A \sqrt{-\det \mathcal{G}}] = 0 \, , $$

where

$$\left(\mathcal{G}^{-1}\right)^{\alpha\beta}_S = \frac{1}{2} \left(\left(\mathcal{G}^{-1}\right)^{\alpha\beta} + \left(\mathcal{G}^{-1}\right)^{\beta\alpha}\right) \quad \left(\mathcal{G}^{-1}\right)^{\alpha\beta}_A = \frac{1}{2} \left(\left(\mathcal{G}^{-1}\right)^{\alpha\beta} - \left(\mathcal{G}^{-1}\right)^{\beta\alpha}\right) \, . \qquad (2.9)$$

On the other hand the equation of motion for tachyon is given by

$$\frac{dV(T)}{dT} e^{-\Phi} \sqrt{-\det \mathcal{G}} - \partial_\alpha [e^{-\Phi} V(T) \partial_\beta T \left(\mathcal{G}^{-1}\right)^{\beta\alpha}_S \sqrt{-\det \mathcal{G}}] = 0 \, . \qquad (2.10)$$

Let us now consider following ansatz

$$X^i = \xi^i, i = 0, \ldots, p, \quad Y = \xi^{p+1}, \quad T = T(y) \qquad (2.11)$$

that corresponds to non-BPS D($p + 1$)-brane spanning the coordinates $x^0, \ldots, x^p$ and y-direction. Note that for this ansatz the components of the matrix $\mathcal{G}$ take the form

$$\mathcal{G}_{ij} = \eta_{ij} + H(\vec{R}, y) \partial_i \vec{R} \partial_j \vec{R} \, ,$$
$$\mathcal{G}_{yy} = H(\vec{R}, y) + H(\vec{R}, y) \partial_y \vec{R} \partial_y \vec{R} + \partial_y T \partial_y T \, ,$$
$$\mathcal{G}_{yi} = \mathcal{G}_{iy} = H(\vec{R}, y) \partial_i \vec{R} \partial_y \vec{R} \, , \qquad (2.12)$$

where we have not specified the form of the world-volume dependence of the modes $\vec{R}$.

It is easy to see that when we insert (2.11) into the equations of motions for X^i (2.8), these equations are satisfied automatically. On the other hand, the equation of motion (2.8) for $X^M \equiv Y$ implies

$$\partial_y e^{-\Phi} V(T) \sqrt{-\det \mathcal{G}} + \frac{1}{2} e^{-\Phi} V(T) \partial_y g_{yy} \left(\mathcal{G}^{-1}\right)^{yy} \sqrt{-\det \mathcal{G}} - $$
$$- \partial_y \left[e^{-\Phi} V(T) g_{yy} \left(\mathcal{G}^{-1}\right)^{yy}_S \sqrt{-\det \mathcal{G}} \right] = 0 \, .$$

Further, the equations of motion for $\vec{R}$ imply

$$\partial_{X^m} e^{-\Phi} V(T) \sqrt{-\det \mathcal{G}} + \tag{2.13}$$
$$+ \frac{1}{2} e^{-\Phi} V(T) (\partial_{x^m} g_{NK} + \partial_{x^m} b_{NK}) \partial_\alpha X^N \partial_\beta X^K \left(\mathcal{G}^{-1} \right)^{\beta\alpha} \sqrt{-\det \mathcal{G}} -$$
$$- \partial_\alpha [e^{-\Phi} V(T) g_{mn} \partial_\beta X^n \left(\mathcal{G}^{-1} \right)_S^{\beta\alpha} \sqrt{-\det \mathcal{G}}] -$$
$$- \partial_\alpha [e^{-\Phi} V(T) b_{mn} \partial_\beta X^n \left(\mathcal{G}^{-1} \right)_A^{\beta\alpha} \sqrt{-\det \mathcal{G}}] = 0 \,,$$

where $m = 7, 8, 9$. To simplify the discussion further let us now presume that $\vec{R}$ is constant. Then

$$\mathcal{G}_{ij} = \eta_{ij}\,, \quad \mathcal{G}_{yy} = H(\vec{R}, y) + (\partial_y T)^2\,, \quad \mathcal{G}_{yi} = 0\,. \tag{2.14}$$

For this ansatz the equations of motion for X^m (2.13) simplifies as

$$\frac{\partial_{x^m} H}{H^{3/2}} \frac{(\partial_y T)^2}{\sqrt{H + (\partial_y T)^2}} = 0\,. \tag{2.15}$$

We see that for $\partial_y T \neq 0$ the only possible solution of the equation above occurs for $\partial_{x^m} H = 0$. It turns out that there is no solutions of this equation for general $\vec{r}_i$. In what follows we restrict ourselves to the case of $\vec{r}_i = 0$. Then it is easy to see that the solution of the equation $\partial_{x^m} H = 0$ is $\vec{R} = 0$.

On the other hand in case of the constant tachyon $T = \mathrm{const}$, we obtain that the equation (2.15) is solved for any $\vec{R} = \vec{c}$ and for general $\vec{r}_i$. This is exactly the same solution that was found recently in [22].

Let us now consider the equation (2.13) in more detail. For (2.11) and for $\vec{R} = 0$ it implies

$$- \frac{\partial_y H(y) V(T)}{2H^{3/2}(y)} \frac{(\partial_y T)^2}{\sqrt{H(y) + (\partial_y T)^2}} - \partial_y \left[\frac{V(T) H^{1/2}(y)}{\sqrt{H(y) + (\partial_y T)^2}} \right] = 0\,, \tag{2.16}$$

where

$$H(y) \equiv \lim_{r \to 0} H = 1 + \frac{k}{2R^2} \frac{1}{1 - \cos y/R}\,. \tag{2.17}$$

Finally, the equation of motion for T (2.10) reduces, for $\vec{R} = 0$, into

$$\frac{dV(T)}{dT} \frac{H^{1/2}(y)}{\sqrt{H(y) + (\partial_y T)^2}} - V(T) \partial_y \left[\frac{\partial_y T}{H^{1/2}(y) \sqrt{H(y) + (\partial_y T)^2}} \right] = 0\,. \tag{2.18}$$

Clearly all these equations have the solution where $\frac{dV}{dT} = 0$ and $T = \mathrm{const}$. For $T_{\max} = 0$ that corresponds to $V(T_{\max}) = 1$ we obtain an unstable D$(p+1)$-brane that wraps y-circle and that is generally localised at an arbitrary $\vec{R} = \vec{c}$ as we have argued above. On the other hand for the minimum of the potential that occurs at $T_{\min} = \pm\infty$, $V(T_{\min}) = 0$ this configuration corresponds to the closed string vacuum after the tachyon condensation.

Our goal is to consider more general solutions. We will argue that in case of the non-BPS D$(p+1)$-brane wrapping y direction there exists solution with natural physical meaning that is however singular. Let us introduce the 'Heaviside step function' $\mathcal{H}(x)$ defined as

$$\mathcal{H}(x) = \begin{cases} 0 & \text{for } x < 0 \\ \frac{1}{2} & \text{for } x = 0 \\ 1 & \text{for } x > 0 \end{cases} . \tag{2.19}$$

The characteristic property of this function is

$$\frac{d}{dx}\mathcal{H}(x) = \delta(x) . \tag{2.20}$$

Now let us consider following form of the tachyon

$$T(y) = -|T_{\min}| + 2|T_{\min}|\mathcal{H}(y - \pi R) . \tag{2.21}$$

The physical meaning of this ansatz is clear. For $0 < y < \pi R$ we have $T = -T_{\min}$; for $y = \pi R$ we have $T = 0$ and for $\pi R < y < 2\pi R$ we have $T = T_{\min}$. It is clear that this tachyon condensation corresponds to Dp-brane that is localised at $y = \pi R$ and to anti Dp-brane that is localised at $y = 0$. More precisely, for this solution we have

$$\frac{dV}{dT} = 0 \tag{2.22}$$

for all y. Explicitly, let us again write equation of motion for the tachyon (2.18)

$$\frac{dV}{dT}\frac{H^{1/2}}{\sqrt{H + (\partial_y T)^2}} - V\partial_y\left[\frac{\partial_y T}{H^{1/2}\sqrt{H + (\partial_y T)^2}}\right] = 0 . \tag{2.23}$$

Firstly, it is clear that the ansatz (2.21) solves the equation of motion for $y \neq 0, \pi R$. For $y = \pi R$, $H(\pi R)$ is finite and can be neglected with respect to $\partial_y T$. Then the term proportional to $\frac{dV}{dT}$ is zero and the second one is equal to

$$V(T=0)\partial_y\left[\frac{\partial_y T}{H^{1/2}\sqrt{H + (\partial_y T)^2}}\right] \approx \frac{\partial_y H}{H^{3/2}} = 0 . \tag{2.24}$$

In the same way we can argue that the ansatz (2.21) solves the equations of motion for $y = 0$ as well. If we insert this ansatz into the non-BPS D$(p+1)$-brane action we obtain

$$S(T(y)) = -\sqrt{2}\mathcal{T}_{p+1}\int_0^{2\pi R} d^{p+1}\xi dy \frac{1}{\sqrt{H(y)}}V(T)\sqrt{H(y) + (\partial_y T)^2}$$

$$= -\sqrt{2}\mathcal{T}_{p+1}|T_{\min}|\int d^{p+1}\xi\left(\frac{1}{\sqrt{H(\pi R)}} + \frac{1}{\sqrt{H(0)}}\right) . \tag{2.25}$$

The result given above suggests that the tachyon solution (2.21) describes the configuration of Dp-brane localised at $y = \pi R$ and anti-Dp-brane localised at $y = 0$.[4] On the other hand

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[4]More precisely, in order to distinguish Dp-brane from anti-Dp-brane we should consider the Wess-Zumino term for a non-BPS Dp-brane [25–27] that is proportional to $\int dT \wedge C$ where C is a collection of Ramond-Ramond forms. The presence of dT determines whether the tachyon kink is Dp-brane or anti Dp-brane.

we see that the solution is singular due to the fact that it is defined using the function $\mathcal{H}$. Further, due to the presence of $|T_{\min}|$ in the final result it is not completely clear that the solution above really describes BPS Dp-branes. On the other hand we present in next section a more natural solution of this problem.

Let us now consider another possibility of the tachyon condensation on non-BPS D$(p+1)$-brane wrapping y-circle and consider following ansatz

$$X^i = \xi^i, \quad i = 0, \ldots, p-1, \quad X^p = \xi^p \equiv x, \quad Y = \xi^{p+1}, \quad T = f(x), \tag{2.26}$$

where $f(u)$ is a function with the properties

$$f'(u) > 0, f(\pm\infty) = \pm\infty, \tag{2.27}$$

and where a is a constant that is taken to infinity in the end.

For the ansatz (2.26) the components of the matrix $\mathcal{G}$ takes the form

$$
\begin{aligned}
\mathcal{G}_{ij} &= \eta_{ij} + H(\vec{R}, y)\partial_i \vec{R} \partial_j \vec{R}, \\
\mathcal{G}_{yy} &= H(\vec{R}, y) + H(\vec{R}, y)\partial_y \vec{R} \partial_y \vec{R}, \\
\mathcal{G}_{yi} &= \mathcal{G}_{iy} = H(\vec{R}, y)\partial_i \vec{R} \partial_y \vec{R}, \\
\mathcal{G}_{pp} &= 1 + H(\vec{R}, y)\partial_x \vec{R} \partial_x \vec{R} + (\partial_x T)^2, \\
\mathcal{G}_{pi} &= \mathcal{G}_{ip} = H(y, \vec{R})\partial_x \vec{R} \partial_i \vec{R}, \\
\mathcal{G}_{yp} &= \mathcal{G}_{py} = H(\vec{R}, y)\partial_y \vec{R} \partial_x \vec{R}.
\end{aligned}
\tag{2.28}
$$

We again solve the equations of motion for constant $\vec{R}$. For this ansatz the matrix $\mathcal{G}$ have following components

$$
\begin{aligned}
\mathcal{G}_{ij} &= \eta_{ij}, \quad \mathcal{G}_{yy} = H(\vec{R}, y), \quad \mathcal{G}_{yi} = \mathcal{G}_{iy} = 0, \\
\mathcal{G}_{pp} &= 1 + (\partial_x T)^2, \quad \mathcal{G}_{pi} = \mathcal{G}_{ip} = \mathcal{G}_{yp} = \mathcal{G}_{py} = 0, \\
\det \mathcal{G} &= -H(\vec{R}, y)(1 + (\partial_x T)^2).
\end{aligned}
\tag{2.29}
$$

Then the equations of motion (2.8) for X^m take the form

$$\frac{\partial_{x^m} H}{H\sqrt{1 + (\partial_x T)^2}}(\partial_x T)^2 = 0 \tag{2.30}$$

that implies that the only solution for $\partial_x T \neq 0$ is $\partial_{x^m} H = 0$ which occurs for $X^m = 0$. The equation of motion for y takes the form

$$-\frac{1}{2gH^{3/2}}\partial_y H V(T)\sqrt{H(1 + (\partial_x T)^2)} + \frac{V(T)\partial_y H}{2gH^{1/2}\sqrt{H(1 + (\partial_x T)^2)}} - \tag{2.31}$$

$$-\partial_y \left[\frac{V(T)H}{2gH^{1/2}H}\sqrt{H(1 + (\partial_x T)^2)} \right] - \partial_x \left[\frac{V(T)H}{2gH^{1/2}\sqrt{H(1 + (\partial_x T)^2)}} \right] =$$

$$= -\frac{1}{2g}\frac{V(T)\partial_y H(\partial_x T)^2}{H\sqrt{1 + (\partial_x T)^2}} - \frac{1}{2g}\partial_x \left[\frac{V(T)}{\sqrt{1 + (\partial_x T)^2}} \right] = 0.$$

Further, the equation of motion for tachyon implies

$$\frac{dV(T)}{dT}\frac{1}{g\sqrt{H}}\sqrt{H(1+(\partial_x T)^2)} - \partial_x\left[\frac{V(T)\partial_x T\sqrt{H}}{g\sqrt{H}\sqrt{1+(\partial_x T)^2}}\right] = 0 \qquad (2.32)$$

and consequently

$$\partial_x\left[\frac{V(T)}{\sqrt{1+(\partial_x T)^2}}\right] = 0 \qquad (2.33)$$

that implies

$$\frac{V(T)}{\sqrt{1+(\partial_x T)^2}} = C \ . \qquad (2.34)$$

Following [24] we can argue that the constant C has to vanish. In fact, for kink solution $T \to \pm\infty$ for $x \to \pm\infty$ where however $\partial_x T$ is finite but $\lim_{T\to\pm\infty} V = 0$. Then we get that $C = 0$ and clearly the tachyon given in (2.26) and in (2.27) implies $C = 0$ in the limit $a \to \infty$. Further, we now argue that the tachyon profile given in (2.26) solves the equation of motion (2.31). Firstly, it is clear that the second term in (2.31) vanishes for (2.34). On the other hand the first term for $a \to \infty$ is proportional to

$$-\frac{1}{2g}\frac{V(T)\partial_y H(\partial_x T)^2}{H\sqrt{1+(\partial_x T)^2}} \approx \frac{V(f(ax))a^2 f'^2(ax)}{|af'(ax)|} \approx \lim_{a\to\infty} e^{-\frac{f(ax)}{\sqrt{2}}}a \to 0 \qquad (2.35)$$

in the limit $a \to \infty$ since we presume that the tachyon potential behaves for large T as $e^{-\frac{T}{\sqrt{2}}}$. Then inserting the ansatz (2.26) into the non-BPS D$(p+1)$-brane action we obtain

$$\begin{aligned}
S &= -\frac{\sqrt{2}\mathcal{T}_{p+1}}{g}\int d^p\xi dy dx V(T(ax))\sqrt{1+(\partial_x T)^2} \\
&\approx -\frac{\sqrt{2}\mathcal{T}_{p+1}}{g}\int dx V(f(ax))af'(ax)\int d^p\xi dy \\
&= -\frac{\sqrt{2}\mathcal{T}_{p+1}}{g}\int dm V(m)2\pi R\int d^p\xi = -\frac{2\pi R\mathcal{T}_p}{g}\int d^p\xi \ ,
\end{aligned} \qquad (2.36)$$

where in the second step we have taken the large a limit and in the last step we have used the fact that [24]

$$\sqrt{2}\mathcal{T}_{p+1}\int dm V(m) = \mathcal{T}_p \ . \qquad (2.37)$$

(2.36) suggests following physical interpretation of the ansatz (2.26): The tachyon kink solution (2.26) can be interpreted as a Dp-brane which is localised at point $x^p = 0$ and is extended in $x^0, \ldots, x^{p-1}, y$. This is precisely the solution presented in [4].

3. Non-BPS D$(p+1)$-brane extended along worldvolume of k NS5-branes

The main goal of our paper is to give an unified description of the configurations presented in [4, 22]. We give a description that is based on the analysis given in [16]. Explicitly, let

us consider a non-BPS D$(p+1)$-brane with their world-volume lying completely along the directions of the NS5-branes. This configuration is achieved with the following ansatz

$$X^\alpha = \xi^\alpha \, , \quad \alpha = 0, \dots, p+1 \tag{3.1}$$

and hence the action for a non-BPS D$(p+1)$-brane takes the form

$$S = -\sqrt{2}\mathcal{T}_{p+1} \int d^{p+2}\xi \frac{1}{\sqrt{H(\vec{R},Y)}} V(T)\sqrt{-\det\mathcal{G}} \, ,$$

$$\mathcal{G}_{\alpha\beta} = \eta_{\alpha\beta} + H(\vec{R},Y)\partial_\alpha\vec{R}\partial_\beta\vec{R} + H(\vec{R},Y)\partial_\alpha Y \partial_\beta Y + \partial_\alpha T \partial_\beta T \, . \tag{3.2}$$

Again solving the equation of motion for constant X^m gives the solution (for coincident NS5-branes)

$$\vec{R} = 0 \, . \tag{3.3}$$

Using this result the action (3.2) simplifies considerably

$$S = -\sqrt{2}\mathcal{T}_{p+1} \int d^{p+2}\xi \frac{1}{\sqrt{H(Y)}} V(T)\sqrt{-\det\mathcal{G}} \, ,$$

$$\mathcal{G}_{\alpha\beta} = \eta_{\alpha\beta} + H(Y)\partial_\alpha Y \partial_\beta Y + \partial_\alpha T \partial_\beta T \, , \tag{3.4}$$

where

$$H(y) = \lim_{r \to 0} H(r,y) = 1 + \frac{k}{4R^2 \sin^2 \frac{Y}{2R}} \, . \tag{3.5}$$

Let us now introduce the geometric tachyon field $\mathcal{T}$ through the relation

$$d\mathcal{T} = \sqrt{H}dY = \sqrt{1 + \frac{k}{4R^2 \sin^2 \frac{Y}{2R}}} dY \, . \tag{3.6}$$

Even if it is possible to explicitly integrate out this equation the result is not well illuminating. On the other hand we can in principle express Y as function of $\mathcal{T}$. Consequently we can introduce the function $W(\mathcal{T})$ defined as

$$W(\mathcal{T}) = \frac{1}{\sqrt{H(\mathcal{T})}} \, . \tag{3.7}$$

We will now discuss basic properties of the potential $W(\mathcal{T})$ defined above. Note that for $Y \to 0$ we get $\mathcal{T} \to \infty$ and consequently from (3.7) we obtain $\lim_{\mathcal{T}\to\infty} W(\mathcal{T}) \to 0$. On the other hand for $Y \to 2\pi R$ we get $\mathcal{T} \to -\infty$ and (3.7) again implies $\lim_{\mathcal{T}\to-\infty} W(\mathcal{T}) \to 0$. Finally for $Y \to \pi R$ we obtain

$$\mathcal{T} \sim \sqrt{1 + \frac{k}{4R^2}}(\pi R - Y) \tag{3.8}$$

and hence for $\mathcal{T} \to 0 (Y \to \pi R)$ we obtain

$$W(\mathcal{T} = 0) = \frac{1}{g\sqrt{1 + \frac{k}{4R^2}}} \, . \tag{3.9}$$

Then the action (3.4)-after performing the field redefinition (3.6)- takes the form

$$S = -\sqrt{2}\mathcal{T}_{p+1} \int d^{p+2}\xi W(\mathcal{T})V(T)\sqrt{-\det\mathcal{G}}\,,$$

$$\mathcal{G}_{\alpha\beta} = \eta_{\alpha\beta} + \partial_\alpha \mathcal{T}\partial_\beta \mathcal{T} + \partial_\alpha T\partial_\beta T\,. \tag{3.10}$$

It will be also useful to calculate the components of the world-volume stress energy tensor from the action (3.10). In order to do this we proceed in the standard way. Explicitly, we replace the flat worldvolume metric $\eta_{\alpha\beta}$ with the general metric $g_{\alpha\beta}$ and define the stress energy tensor as

$$T_{\alpha\beta} = -\frac{2}{\sqrt{-g}}\frac{\delta S}{\delta g^{\alpha\beta}}\,. \tag{3.11}$$

Then from (3.10) we obtain that $T_{\alpha\beta}$ are equal to

$$T_{\alpha\beta} = -\sqrt{2}\mathcal{T}_{p+1}V(T)W(\mathcal{T})\eta_{\alpha\gamma}\eta_{\beta\delta}\left(\mathcal{G}^{-1}\right)^{\delta\gamma}\sqrt{-\det\mathcal{G}} \tag{3.12}$$

using

$$\left.\frac{\delta g_{\gamma\delta}}{\delta g^{\alpha\beta}}\right|_{g_{\alpha\beta}=\eta_{\alpha\beta}} = -\eta_{\gamma\alpha}\eta_{\beta\delta}\,. \tag{3.13}$$

Now using also the fact that the world-volume theory defined by the action (3.10) is Poincare invariant we obtain that the components of the stress energy tensor obey the conservation laws

$$\partial_\alpha \eta^{\alpha\gamma}T_{\gamma\beta} = 0\,. \tag{3.14}$$

Note also that this is an important difference with respect to the situation studied in the previous section where due to the explicit dependence of the world-volume theory on y the components of the stress energy tensor do not obey the conservation equation (3.14).

Let us now consider the solution where the tachyon T is in its unstable minimum $T = 0$ while $\mathcal{T}$ depends on one spatial coordinate, say $\xi^{p+1} = x$. Then (3.14) implies

$$\partial_x T_{xx} = 0\,. \tag{3.15}$$

Since for $x \to \pm\infty$ we presume $\mathcal{T} \to \pm\infty$ and consequently $W \to 0$ we obtain that T_{xx} has to vanish everywhere. This is well known result given in [24]. Explicitly, following [24] we can consider $\mathcal{T}$ in the form

$$\mathcal{T} = f(ax)\,, \tag{3.16}$$

where $f(u)$ is a function with the properties

$$f'(u) > 0\,, f(\pm\infty) = \pm\infty\,, \tag{3.17}$$

and where a is a constant that is taken to infinity in the end. Then it is easy to see that T_{xx} vanishes for all x in the limit $a \to \infty$ since

$$T_{xx} = -\sqrt{2}\mathcal{T}_{p+1}\frac{W(\mathcal{T})}{\sqrt{1+(\partial_x\mathcal{T})^2}} \approx -\sqrt{2}\mathcal{T}_{p+1}\frac{W(f(ax))}{a|f'(ax)|}\,. \tag{3.18}$$

It is clear that this expression vanishes in the limit $a \to \infty$ where $W \to 0$ and $a \to \infty$. This expression also vanishes for $x = 0$ since now $W(0)$ is finite but $a \to \infty$. Let us now calculate the components of the kink stress energy tensor. Again, following [24] we define these components as

$$
\begin{aligned}
T_{ij}^{kink} &= \int dx T_{ij} \\
&= -\eta_{ij}\sqrt{2}\mathcal{T}_{p+1} \int dx W(f(ax))a|f'(ax)| \\
&= -\eta_{ij}\sqrt{2}\mathcal{T}_{p+1} \int_{\infty}^{-\infty} dm W(m) \\
&= -\eta_{ij}\sqrt{2}\mathcal{T}_{p+1} \int_0^{2\pi R} dy \\
&= -\eta_{ij}\sqrt{2}\mathcal{T}_{p+1} 2\pi R \\
&= -\eta_{ij}\sqrt{2}\mathcal{T}_p R \,,
\end{aligned}
\tag{3.19}
$$

where $i, j = 0, \ldots, p$. In other words the tachyon kink given above corresponds to non-BPS Dp-brane that is sitting for $(x < 0)$ on the world-volume of k-NS5-branes, and then wraps the circle y of radius $2\pi R$ at $x = 0$ and then is again sitting for $x > 0$. We have seen that the tension of the configuration corresponds to non-BPS D$(p + 1)$-brane wrapped y-circle and sitting at $\vec{R} = 0$ in the same way as in paper [22].

We can also consider another solution given by the ansatz

$$
\mathcal{T} = \text{const.}, \quad T = T(x) \,.
\tag{3.20}
$$

Again the equation of motion for $\mathcal{T}$ implies that $\mathcal{T} = 0$ or $\mathcal{T} = \pm\infty$. We will consider the ansatz $\mathcal{T} = 0$ corresponding to non-BPS D$(p + 1)$-brane localised at the point $y = \pi R$. Then we will construct the tachyon singular solution exactly as in [24]. Explicitly, let us consider the tachyon profile

$$
T(x) = f(ax) \,,
\tag{3.21}
$$

where f has the same properties as in (3.17). The invariance of the world-sheet energy tensor implies that

$$
\begin{aligned}
\partial_x T_{xx} &= 0 \,, \\
T_{xx} &= -\sqrt{2}\mathcal{T}_{p+1}W(0)V(T)\left(\mathcal{G}^{-1}\right)^{xx}\sqrt{-\det\mathcal{G}} \\
&= -\sqrt{2}\mathcal{T}_{p+1}W(0)V(f(ax))\frac{1}{\sqrt{1 + a^2 f'^2(ax)}} \\
&\approx -\sqrt{2}\mathcal{T}_{p+1}W(0)\frac{V(f(ax))}{a|f'(ax)|} \,.
\end{aligned}
\tag{3.22}
$$

Again standard arguments imply that the T_{xx} has to vanish. On the other hand the

remaining components of the stress energy tensor T_{ij} are equal to

$$T_{ij} = -\eta_{ij}\sqrt{2}\mathcal{T}_{p+1}W(0)V(f(ax))\sqrt{1 + a^2f'^2(ax)}$$
$$\approx -\eta_{ij}\sqrt{2}\mathcal{T}_{p+1}\frac{1}{g\sqrt{1 + \frac{k}{4R^2}}}V(f(ax))a|f'(ax)| \ . \tag{3.23}$$

We again define the stress energy tensor of the kink as

$$T_{ij}^{kink} = \int dx T_{ij}(x)$$
$$= -\eta_{ij}\sqrt{2}\mathcal{T}_{p+1}\frac{1}{g\sqrt{1 + \frac{k}{4R^2}}}\int dx V(f(ax))a|f'(ax)|$$
$$= -\eta_{ij}\sqrt{2}\mathcal{T}_{p+1}\frac{1}{g\sqrt{1 + \frac{k}{4R^2}}}\int dm V(m)$$
$$= -\eta_{ij}\frac{\mathcal{T}_p}{g\sqrt{1 + \frac{k}{4R^2}}} \tag{3.24}$$

using (2.37). The physical interpretation of the solution (3.20) is as follows. It corresponds to BPS Dp-brane that is localised at the point $y = \pi R, \vec{R} = 0$. Again, the tension of this geometrically unstable Dp-brane agrees exactly with the results given in [4, 22].

4. Small radius limit

Following [4, 22] we now consider the limit

$$R \to 0, \quad g \to 0, \quad \tilde{g} \equiv \frac{g}{R} = \text{fixed} \tag{4.1}$$

and define new coordinate

$$\tilde{y} = \frac{y}{R}, \quad \vec{\tilde{r}} = \frac{\vec{r}}{R} \ . \tag{4.2}$$

In this limit the NS5-brane background takes the form

$$ds^2 = \eta_{\mu\nu}dx^\mu dx^\nu + h(\vec{\tilde{r}}, \tilde{y})(d\tilde{y}^2 + d\vec{\tilde{r}}^2), \quad e^{2\Phi} = \tilde{g}^2 h(\vec{\tilde{r}}, \tilde{y}), \tag{4.3}$$

where

$$h(\vec{\tilde{r}}, \tilde{y}) = \frac{k}{2\tilde{r}}\frac{\sinh \tilde{r}}{\cosh \tilde{r} - \cos \tilde{y}}, \quad \tilde{r} \equiv |\vec{\tilde{r}}| \ . \tag{4.4}$$

In this coordinate system the BPS Dp-brane in unstable equilibrium is situated at $\vec{\tilde{r}} = 0, \tilde{y} = \pi$. The formulae for the tension and the tachyon mass of this G-type brane takes the form [4, 22]

$$\tau_p = \frac{2}{\sqrt{k}}\tilde{g}^{-1}\mathcal{T}_p, \quad m_T^2 = -\frac{1}{k} \ . \tag{4.5}$$

Let us now discuss an unstable D$(p + 1)$-brane in the geometry (4.3) where we begin with non-BPS D$(p + 1)$-brane wrapped $\tilde{y}$ direction. We restrict to the case of the constant

$\tilde{X}^m = 0$ that is necessary in case when $\partial_{\tilde{y}} T \neq 0$. Let us consider the equation of motion for $\tilde{Y} = \xi^{p+1}$

$$-\frac{\partial_{\tilde{y}} h}{h^{3/2}} V(T) \frac{\partial_{\tilde{y}} T}{\sqrt{h + (\partial_{\tilde{y}} T)^2}} - \partial_{\tilde{y}} \left[\frac{V(T) h^{1/2}}{\sqrt{h + (\partial_{\tilde{y}} T)^2}} \right] = 0 \qquad (4.6)$$

and consider following ansatz for tachyon

$$T(\tilde{y}) = -|T_{\min}| + 2|T_{\min}| \mathcal{H}(\tilde{y} - \pi) . \qquad (4.7)$$

Following arguments given in the second section we can easily shown that $T(\tilde{y})$ solves the equation of motions. Inserting (4.7) into a non-BPS D(p+1)-brane action we get

$$\begin{aligned} S &= -\frac{\sqrt{2} \mathcal{T}_{p+1}}{\tilde{g}} \int d^{p+1} \xi d\tilde{y} V(T) \frac{1}{\sqrt{h}} \sqrt{h + (\partial_{\tilde{y}} T)^2} \\ &= -\frac{\sqrt{2} \mathcal{T}_{p+1}}{\tilde{g}} |T_{\min}| \int d^{p+1} \xi \left(\frac{1}{\sqrt{h(\pi)}} + \frac{1}{\sqrt{h(0)}} \right) . \end{aligned} \qquad (4.8)$$

In the same way as in the second section, it is tempting to interpret the resulting configuration as a a BPS Dp-brane anti-Dp-brane localised at $\tilde{y} = \pi$ and $\tilde{y} = 0$ respectively. However as in the second section we see that this interpretation is not well precise due to the presence of the factor $|T_{\min}| = \infty$. In fact, in order to derive more natural solution we proceed in the same way as in the previous section. Let us again consider the non-BPS D$(p + 1)$-brane that is embedded in $R^{5,1}$ in the geometry (4.3). This can be achieved with the choice of the gauge

$$X^\alpha = \xi^\alpha , \quad \alpha = 0, \ldots, p + 1 . \qquad (4.9)$$

In this case the non-BPS D$(p + 1)$-brane action takes the form

$$S = -\frac{\sqrt{2} \mathcal{T}_{p+1}}{\tilde{g}} \int d^{p+2} \xi \frac{V(T)}{\sqrt{h(\tilde{R}, \tilde{Y})}} \sqrt{-\det \mathcal{G}} , \qquad (4.10)$$

where

$$\mathcal{G}_{\alpha\beta} = \eta_{\alpha\beta} + h(\tilde{R}, \tilde{Y}) \left(\partial_\alpha \vec{\tilde{R}} \partial_\beta \vec{\tilde{R}} + \partial_\alpha \tilde{Y} \partial_\beta \tilde{Y} \right) + \partial_\alpha T \partial_\beta T . \qquad (4.11)$$

The action (4.10) describes the non-BPS D(p+1)-brane that is localised in the transverse space labelled with $\vec{\tilde{R}}, \tilde{Y}$. As in the case of a BPS Dp-brane studied in [4, 22] we will be interested in the study of the dynamics of the mode $\tilde{Y}$. In other words we firstly solve the equations of motion for $\vec{\tilde{R}}$ and search the solutions where $\vec{\tilde{R}}$ are constant. In fact, as follows from the equations of motion for $\tilde{X}^m$ we obtain that they are obeyed for

$$\tilde{X}^m = 0 . \qquad (4.12)$$

In what follows we restrict to the study of the action for the $\tilde{Y}$ and T only. Note that for $\tilde{R} = 0$ $h(\tilde{R}, \tilde{Y})$ is equal to

$$h(\tilde{Y}, 0) \equiv h(\tilde{Y}) = \frac{k}{4 \sin^2 \frac{\tilde{Y}}{2}} . \qquad (4.13)$$

Then the non-BPS D$(p+1)$-brane action takes the form

$$S = -\frac{\sqrt{2}\mathcal{T}_{p+1}}{\tilde{g}} \int d^{p+2}\xi \frac{V(T)}{\sqrt{h(\tilde{Y})}} \sqrt{-\det(\eta_{\alpha\beta} + h(\tilde{Y})\partial_\alpha\tilde{Y}\partial_\beta\tilde{Y} + \partial_\alpha T\partial_\beta T)} \,, \qquad (4.14)$$

where $V(T)$ is equal to

$$V(T) = \frac{1}{\cosh\frac{T}{\sqrt{2}}} \,. \qquad (4.15)$$

Now, following [3, 4] we introduce tachyon field $\mathcal{T}$ that is related to $\tilde{Y}$ through the relation

$$\frac{d\mathcal{T}}{d\tilde{Y}} = \sqrt{h(\tilde{Y})} = \frac{\sqrt{k}}{2\sin\frac{\tilde{Y}}{2}} \,. \qquad (4.16)$$

This differential equation has the solution

$$\cos\frac{\tilde{Y}}{2} = \frac{\sinh\frac{\mathcal{T}}{\sqrt{k}}}{\cosh\frac{\mathcal{T}}{\sqrt{k}}} \,, \qquad (4.17)$$

where we have used the boundary condition that for $\tilde{Y} = \pi$ the tachyon field $\mathcal{T}$ is equal to zero. Note also that (4.17) implies that for $\tilde{Y} \to 0$ $\mathcal{T} \to \infty$ while for $\tilde{Y} \to 2\pi$ we obtain that $\mathcal{T} \to -\infty$. In other words the new tachyon field belongs to the interval $\mathcal{T} \in (-\infty, \infty)$. Then we obtain

$$h(\tilde{Y}(\mathcal{T})) = \frac{k}{4}\cosh^2\frac{\mathcal{T}}{\sqrt{k}} \qquad (4.18)$$

and hence the tachyon effective action (4.14) can be written as

$$S = -\tau_{p+1} \int d^{p+2}\xi \mathcal{V}(T,\mathcal{T}) \sqrt{-\det(\eta_{\alpha\beta} + \partial_\alpha\mathcal{T}\partial_\beta\mathcal{T} + \partial_\alpha T\partial_\beta T)} \,,$$
$$\mathcal{V}(\mathcal{T}, T) = \frac{1}{\cosh\frac{\mathcal{T}}{\sqrt{k}}\cosh\frac{T}{\sqrt{2}}} \equiv V\left(\mathcal{T}, \frac{1}{k}\right) V\left(T, \frac{1}{2}\right) \,, \qquad (4.19)$$

where

$$\tau_{p+1} = \frac{2\sqrt{2}\mathcal{T}_{p+1}}{\sqrt{k}\tilde{g}} \,, \quad V(f,x) \equiv \frac{1}{\cosh(f\sqrt{x})} \,. \qquad (4.20)$$

Now we come to the study of some solutions of the equations of motion for T and $\mathcal{T}$ that arise from the action (4.19). Note also that these equations has been also discussed in [16].

Let us start with the solution that describes non-BPS D$(p+1)$-brane wrapped transverse circle. As the first step we determine the equations of motion for $\mathcal{T}, T$ from (4.19)

$$-\frac{\sinh\frac{\mathcal{T}}{\sqrt{k}}}{\sqrt{k}\cosh^2\frac{\mathcal{T}}{\sqrt{k}}\cosh\frac{T}{\sqrt{2}}} \sqrt{-\det\mathcal{G}} - \partial_\alpha\left[\frac{1}{\cosh\frac{\mathcal{T}}{\sqrt{k}}\cosh\frac{T}{\sqrt{2}}}\partial_\beta\mathcal{T}\left(\mathcal{G}^{-1}\right)^{\beta\alpha}\sqrt{-\det\mathcal{G}}\right] = 0 \,,$$

$$-\frac{\sinh\frac{T}{\sqrt{2}}}{\sqrt{2}\cosh\frac{\mathcal{T}}{\sqrt{k}}\cosh^2\frac{T}{\sqrt{2}}} \sqrt{-\det\mathcal{G}} - \partial_\alpha\left[\frac{1}{\cosh\frac{\mathcal{T}}{\sqrt{k}}\cosh\frac{T}{\sqrt{2}}}\partial_\beta T\left(\mathcal{G}^{-1}\right)^{\beta\alpha}\sqrt{-\det\mathcal{G}}\right] = 0 \,. \quad (4.21)$$

It is easy to see that for $k = 2$ both the equations are essentially the same. Let us now consider the situation when $\mathcal{T}$ is a function of one spatial variable on non-BPS D$(p + 1)$-brane, say $\xi^{p+1} \equiv x$ and construct the singular kink following the analysis performed in [24]. First of all, the equations of motion (4.21) for $T = \text{const}$ implies that we have two solutions $T = T_{\min}$ or $T = T_{\max}$. We consider the solution $T = T_{\max}$ corresponding to unstable D$(p + 1)$-brane. Then the equation of motion for $\mathcal{T} = \mathcal{T}(x)$ takes the form

$$- \frac{\sinh \frac{\mathcal{T}}{\sqrt{k}} \sqrt{1 + (\partial_x \mathcal{T})^2}}{\sqrt{k} \cosh^2 \frac{\mathcal{T}}{\sqrt{k}}} - \partial_x \left(\frac{1}{\cosh \frac{\mathcal{T}}{\sqrt{k}}} \frac{\partial_x \mathcal{T}}{\sqrt{1 + (\partial_x \mathcal{T})^2}} \right) = 0 \tag{4.22}$$

that can be written as

$$\partial_x \left(\frac{1}{\cosh \frac{\mathcal{T}}{\sqrt{k}} \sqrt{1 + (\partial_x \mathcal{T})^2}} \right) = 0 \ . \tag{4.23}$$

In other words the expression in the bracket above does not depend on x. Since for a kink solution $\mathcal{T} \to \pm\infty$ as $x \to \pm\infty$ and $V(\mathcal{T}, k^{-1}) \to 0$ in this limit we obtain that the expression in the bracket vanishes for $x \to \infty$ and from its independence on x it implies that it vanishes everywhere. This in turn implies that we should have

$$\mathcal{T} = \pm\infty \text{ or } \partial_x \mathcal{T} = \infty \text{ (or both) for all } x \ . \tag{4.24}$$

Clearly this solution looks singular. Again, following [24] we consider the field configuration

$$\mathcal{T}(x) = f(ax), \quad f(u) = -f(-u), \quad f'(u) < 0 \quad \forall x, \quad f(\pm\infty) = \mp\infty \tag{4.25}$$

that in the limit $a \to \infty$ is singular. For this solution however we get that

$$\frac{V(f(ax), k^{-1})}{\sqrt{1 + (\partial_x \mathcal{T})^2}} \approx \frac{V(f(ax), k^{-1})}{a|f'(ax)|} \tag{4.26}$$

that vanishes everywhere at the limit $a \to \infty$ since the numerator vanishes (except at $x = 0$) and the denominator blows up everywhere.

For next purposes it will be also useful to calculate the stress energy tensor from the action (4.19). Following the analysis outlined in previous section we obtain

$$T_{\alpha\beta} = -\tau_{p+1} \mathcal{V}(T, \mathcal{T}) \eta_{\alpha\gamma} \eta_{\beta\delta} \left(\mathcal{G}^{-1} \right)^{\delta\gamma} \sqrt{-\det \mathcal{G}} \ . \tag{4.27}$$

Then we obtain following components of the stress energy tensor

$$\begin{aligned} T_{ij}(x) &= -\eta_{ij} \tau_{p+1} V(\mathcal{T}, k^{-1}) \sqrt{1 + (\partial_x \mathcal{T})^2} \\ &= -\eta_{ij} \tau_{p+1} V(f(ax), k^{-1}) \sqrt{1 + a^2 f'^2(ax)} \\ &\approx -\eta_{ij} \tau_{p+1} V(f, k^{-1}) |af'(ax)| \qquad i, j = 0, \ldots, p \end{aligned} \tag{4.28}$$

in the limit $a \to \infty$. Then the integrated T_{ij} associated with the codimension one solution are equal to

$$\begin{aligned} T_{ij}^{kink} &= \int dx T_{ij} \\ &= -\eta_{ij} \tau_{p+1} \int dx V \left(f, \frac{1}{k} \right) |af'(ax)| = -\eta_{ij} \tau_{p+1} \int dm V \left(m, \frac{1}{k} \right) , \end{aligned} \tag{4.29}$$

where $m = f(ax)$. Thus T_{ij}^{kink} depend on V and not on the form of $f(u)$. Note that for $V(f, k^{-1})$ defined in (4.20) we obtain

$$\int dm \frac{1}{\cosh \frac{m}{\sqrt{k}}} = \sqrt{k} \frac{(2\pi)}{2} .$$ (4.30)

In fact, using (4.30) we obtain that the tension of the resulting kink is equal to

$$\tau_{p+1} \sqrt{k} \frac{(2\pi)}{2} = 2\pi \sqrt{2} \mathcal{T}_{p+1} \tilde{g}^{-1} = \sqrt{2} \tilde{g}^{-1} \mathcal{T}_p .$$ (4.31)

Let us give the geometrical meaning of this solution. Since $\mathcal{T}$ is directly related to the coordinate $\tilde{y}$ that parameterises the position of a non-BPS D($p+1$)-brane on the transverse circle, the singular kink solution corresponds to non-BPS D($p+1$)-brane that sits on the top of the five branes for all $x < 0$, at $x = 0$ wraps the $\tilde{y}$ circle and then back to the five branes at $\tilde{y} = 2\pi$ where it stays for all $x > 0$. In other words this solution describes non-BPS Dp-brane wrapping around the transverse circle that was recently discussed in [22].

Let us now consider following ansatz

$$\mathcal{T} = \text{const}, \quad T(x) = f(ax) .$$ (4.32)

Firstly, the solutions of the equation of motion for constant $\mathcal{T}$ is either $\mathcal{T} = \pm\infty$ or $\mathcal{T} = 0$. The case $\mathcal{T} = \pm\infty$ corresponds to non-BPS D($p + 1$)-brane sitting on world-volume of NS5-branes. On the other hand the case $\mathcal{T} = 0$ corresponds to non-BPS D($p + 1$)-brane sitting at the point $\tilde{Y} = \pi$. Let us consider this situation where now $V\left(\mathcal{T}, \frac{1}{k}\right) = 1$. Then the invariance of the world-sheet energy tensor implies that

$$\begin{aligned}
\partial_x T_{xx} &= 0 , \\
T_{xx} &= -\tau_{p+1} \mathcal{V}(\mathcal{T}, T) \left(\mathcal{G}^{-1}\right)^{xx} \sqrt{-\det \mathcal{G}} \\
&= -\tau_{p+1} V\left(f(ax), \frac{1}{2}\right) \frac{1}{\sqrt{1 + a^2 f'^2(ax)}} \\
&\approx -\tau_{p+1} \frac{V(f(ax), \frac{1}{2})}{a|f'(ax)|} .
\end{aligned}$$ (4.33)

Again standard arguments imply that the T_{xx} has to vanish. On the other hand the remaining components of $T_{\alpha\beta}$ are equal to

$$\begin{aligned}
T_{ij} &= -\eta_{ij} \tau_{p+1} V\left(f(ax), \frac{1}{2}\right) \sqrt{1 + a^2 f'^2(ax)} \\
&\approx -\eta_{ij} \tau_{p+1} V\left(f(ax), \frac{1}{2}\right) a|f'(ax)| .
\end{aligned}$$ (4.34)

We define the stress energy tensor of the kink as the integration of (4.34) over x

$$\begin{aligned}
T_{ij}^{kink} &= \int dx T_{ij}(x) \\
&= -\eta_{ij} \tau_{p+1} \int dx V\left(f(ax), \frac{1}{2}\right) a|f'(ax)| \\
&= -\eta_{ij} \tau_{p+1} \int dm V\left(m, \frac{1}{2}\right) = -\eta_{ij} \frac{2\mathcal{T}_p}{\sqrt{k}} \tilde{g}^{-1}
\end{aligned}$$ (4.35)

using

$$\sqrt{2}\mathcal{T}_{p+1}\int dm V\left(m,\frac{1}{2}\right) = \mathcal{T}_p \ . \tag{4.36}$$

The geometrical meaning of this solution is clear. It is the G-type Dp-brane sitting at the point $\tilde{y} = \pi$.

In summary, we have shown that non-BPS D$(p + 1)$-brane that is stretched along the world-volume directions of the NS5-brane contains as its solution the non-BPS D$(p + 1)$-brane that wraps the transverse circle and also contains solution corresponding to G-type D-brane that sits at the point $\tilde{y} = \pi$. Note also that for $k = 2$ the fields $\mathcal{T}$ and T have completely the same dynamics. Since these two D-branes can be interpreted as solutions of one world-volume theory they are-from the point of view of this theory-indistinguishable. In other words, for $k = 2$, the unstable D$(p + 1)$-brane wrapping transverse circle and the G-type Dp-brane sitting at the point $\tilde{y} = \pi$ are actually the same object. We would like to mention that this fact can serve as a support of the Sen's recent conjecture.

5. Conclusion and discussion

In this paper we have studied the non-BPS branes from the view point of DBI analysis in NS5-brane background. We have presented two complimentary description of the non-BPS D-brane in NS 5-brane background depending on whether the brane wraps the transverse circle of extended along the NS5-brane worldvolume directions. We have shown that when the non-BPS D-brane wraps the transverse circle along the NS 5-brane, there exist tachyon solutions that reproduce the unstable D-brane solutions of [22]. We have also argued that the physical interpretation is not completely clear due to the presence of the infinite factor corresponding to the value of the tachyon at its minimum.

The second description has the non-BPS branes stretched along the longitudinal directions on the NS 5-branes. The main advantage of this approach with respect to the previous one is that now the mode that parametrises the position of non-BPS D$(p + 1)$-brane along transverse circle can be mapped to geometric tachyon field $\mathcal{T}$ that has similar properties as the ordinary one T. Then it is easy to construct the solution given in [24]. Explicitly, we have shown that this theory contains a non-BPS D$(p + 1)$-brane that wraps the transverse circle (y), along with a solution which can be seen as a Dp-brane with a geometrical instability (the G-type D-brane) localised at $y = \pi R$. This is a hint of the unified description of the G-type D-brane and the U-type D-brane arising out of the worldvolume theory on the non-BPS D$(p + 1)$-brane localised on y-circle. Then we have studied the situation in the small radius limit. We show that the correspondence between the G-type and U-type branes is also obeyed. We have further shown the equivalence of the geometric tachyon field and the usual open string tachyon field and have shown that for the value $k = 2$ they essentially have the same dynamics. For this value the unstable D-brane which wraps the transverse circle and the G-type D-brane which sitting at $\tilde{y} = \pi$ are actually the same object.

We would also like to suggest an extension of the present work. Firstly, it would be interesting to study the properties of the resulting solutions from the Hamiltonian point

of view. In fact, it is well known from the Hamiltonian analysis of the unstable Dp-branes that many new interesting phenomena occur, for example the emergence of the closed strings [28 – 35]. It would be extremely interesting to see related phenomena in case of geometric tachyon and find their physical interpretation. We hope to return to this problem in future.

Before ending the paper, we would like to mention few things about the dual ALF theory as discussed in [22]. The non-BPS D$(p+1)$-brane of the original theory along $x^0, \ldots, x^p, y$ placed at $\vec{r}$ are easy to describe in the dual theory. This goes over to a non-BPS Dp-brane along $x^0, \ldots, x^p$ in the dual system placed at fixed values of $\vec{r}$ and ψ in the ALF space, with the location along ψ determined by the Wilson line along y of the original system. On the other hand let us consider non-BPS D$(p+1)$-brane in the original theory that is sitting at particular point in y. In the dual theory it wraps ψ circle. However we immediately come to the puzzle that has the same origin as in case of BPS Dp-brane. Namely, G-type unstable D-branes that in the original description correspond by placing BPS Dp-brane along $x^0, \ldots, x^p$ at $(\vec{r}, y = 0)$ or $(\vec{r} = 0, y = \pi R)$. Since T-duality acting on a D-brane localised at a point on a circle maps it to a D-brane wrapped along the dual circle we expect that the T-dual description of the G-type brane is BPS D$(p+1)$-brane along $x^0, \ldots, x^p$ and ψ and placed at $\vec{r} = 0$. The coordinate y of the original Dp-brane corresponds to the Wilson line along ψ on the dual D$(p+1)$-brane. However at the level of supergravity approximation we do not see a potential for the Wilson line. However they are expected to be induced by the world-sheet instanton corrections [36 – 38]. It would be again extremely interesting whether we can find field redefinition that can maps this potential and the Wilson line variable to the tachyon-like form.

Acknowledgments

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